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Soft-target dispatch strategies for transit services with uncertain passenger arrivals

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ABSTRACT

Conventional transit systems, operated based on pre-defined schedules, are prone to oversaturation and undersaturation caused by stochastic fluctuations in demand. To address this issue, we propose adaptive dispatch strategies, namely the 'soft-target' models, to dispatch vehicles when the number of passengers arrived or requests received meets an occupancy target. The occupancy target is designed to decline gradually as time elapses since the last dispatch. The proposed models are optimised to minimise the generalised system cost, comprising both the costs of passengers and the operator. Simulation tests and numerical experiments indicate that the proposed strategies outperform their counterparts (e.g. the 'hard-target' model, where the occupancy target is fixed, and the existing soft-target model) when the demand level is low and variation is large. The proposed models have broad potential applications in non-reservation-based transit systems, such as off-peak conventional transit lines and spontaneous on-demand mobility.

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Public transit; adaptive dispatching; soft-target dispatch strategy; uncertain passenger arrivals; optimisation; capacity constraint

1. Introduction

One of the key operational decisions in public transportation is the timing of vehicle dispatches. Conventional transit systems dispatch vehicles according to predetermined schedules (Ceder 2016; Daganzo and Ouyang 2019). Extensive models have been developed to generate and optimise transit schedules in terms of headways (or frequencies) (Farrando et al. 2025; Gkiotsalitis and Cats 2017, 2022; Huang et al. 2017; Hurdle 1973; Jiang 2022; Newell 1971; Tan et al. 2024; W. Fan and Machemehl 2008; Z. Chen, Li, and Zhou 2020; Z. Zhang et al. 2025) and timetables (Bie et al. 2021; Hu, Fu, and Feng 2023; Liu, Ceder, and Chowdhury 2017; Silva-Soto and Ibarra-Rojas 2021; Wu et al. 2016;

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X. Guo et al. 2017; Y. Wang et al. 2023; Y. Yin et al. 2019; Z. Chen, Li, and Zhou 2019; Zhao, Sun, and Cats 2023; Zhou et al. 2025; Zhu et al. 2024), as well as some hybrid service optimisation studies based on both (Eltved, Nielsen, and Rasmussen 2019; Jiang, Kjaer Rasmussen, and Anker Nielsen 2022; X. Chen et al. 2025).

These schedule-based dispatch approaches usually rely on historical or predicted demand data but lack the consideration of real-time actual demand that fluctuates with uncertainty, leading to operational inefficiencies in terms of oversaturation and undersaturation. The consequence is significant in non-reservation-based transit services, such as regular buses, bus rapid transit (BRT), and subways, which constitute the primary public transportation modes. In some instances, these transit systems become overcrowded at stations and in vehicles due to sudden influxes of mass passengers or the accumulation of those who failed to board (J. Yang et al. 2017; Q. Guo et al. 2023). Conversely, when there are unexpectedly few passenger arrivals, transit vehicles may be underutilised, resulting in buses and trains running with low occupancy or even empty.

Regarding demand uncertainty, some early classic studies optimise service frequency and fleet size for different periods (Jansson 1980) or utilise stop-skip strategies (Eberlein, Wilson, and Bernstein 1999; Furth and Brian Day 1985) to address stochastic demand during one day. Although these research results are easily implementable in actual operations, they lack flexibility when dealing with arriving passengers in real-time. In response to this, recent studies have employed stochastic optimisation approaches for transit service scheduling. For instance, Hadas and Shnaiderman (2012) proposed a cost-based frequency setting model to optimise the headway and vehicle capacity in bus corridors considering running empty-seats and stochastic demand. Tian, Lin, and Wang (2021) applied a mixed-integer stochastic programming model to optimise fleet size and service allocation for autonomous and conventional buses, leveraging the Sample Average Approximation (SAA) method to handle demand uncertainty. X. Guo et al. (2024) focussed on the transit frequency setting problem using a robust optimisation (RO) model to minimise worst-case scenario costs, employing dimensionality reduction techniques to handle large-scale networks. Additionally, a robust path recommendation model for public transit disruptions combined simulation-based methods with robust optimisation to minimise system-wide travel time under demand uncertainty (Mo et al. 2023). Metaheuristic models that incorporate stochastic travel time and demand have also enhanced schedule reliability (Cacchiani, Qi, and Yang 2020). However, these stochastic optimisation models are often complex, demanding extensive model construction and advanced solution techniques, which hinders their practical applications.

Adaptive dispatching models offer an alternative option for addressing the issue of extraordinary oversaturation and undersaturation. By using straightforward dispatching criteria, they dispatch vehicles in response to real-time demand. The crucial decision involves dynamically determining when to dispatch vehicles, to deliver the on-board passengers, or to pick up the received requests, while accounting for the uncertainty of newcomers.

Studies on adaptive dispatch models are still in the early stages and have limitations that impede wide applications (H. Zhang 2026; Markovska, Ruseva, and Kabaivanov 2022). For instance, Amirgholy and Gonzales (2016) described a strategy that dispatches vehicles when the number of received requests meets the vehicle capacity. As a result, the transit system avoids unfavourable over- and under-saturation completely. Consequently, the work of finding optimal vehicle capacity and scheduling strategy has received considerable attention (Cats and Glück 2019; Jara-Díaz, Fielbaum, and Gschwender 2017). Mahmoudi, Saidi, and Wirasinghe (2025) proposed a joint optimisation method for vehicle capacity and headway considering the over-saturation situation. To meet the fluctuating demands, some studies have introduced modular vehicles with variable capacity into the transportation system (J. Wang et al. 2026; Tang et al. 2024; Y. Wang et al. 2025). Gaborit et al. (2026) develops an adaptive large neighbourhood search based matheuristic for an acyclic bus timetabling problem with time-dependent travel time and demand data.

The real-time control methods, such as reinforcement learning (RL), have also been adopted (A. Chang, Song, and Wang 2024; Galarza Montenegro, Sörensen, and Vansteenwegen 2025; Gavriilidou and Cats 2019; Pan et al. 2026; Su et al. 2024). Chow, Li, and Ying (2024) presented an adaptive scheduling model that utilises RL based on real-time passenger demand. Their study concentrates on an over-saturated transit system subject to total fleet size and vehicle capacity constraints. Rather than setting the occupancy target as the vehicle capacity, Daganzo and Ouyang (2019) suggested that optimisation could be made by considering the trade-off between passengers' waiting time and the operator's operational cost. W. Fan, Gu, and Xu (2024) proposed an optimal design model to optimise the occupancy target of ride-pooling vehicles considering spatial heterogeneity (that arises from non-uniform demand and different travel distances). The pre-specified or optimised occupancy targets, called the 'hard-target' strategy, are fixed and not responsible for uncertainty in actual demand. Under the hard-target strategies, the arrived passengers may experience excessively long waiting delays due to the unexpected late arrivals of the last few ones.

1.1. Research gaps and motivation

To eliminate such prolonged waiting time, Daganzo and Ouyang (2019) proposed a 'soft-target' strategy that adopts a time-declining target (hereafter termed the D-O model or the original D-O model to be distinguished from ours), which decreases gradually toward zero and will dispatch vehicles in a limited time interval. The D-O model has been applied to ride-pooling as on-demand feeder services (W. Fan et al. 2025). Nevertheless, there exist some flaws in the D-O model, as revisited in Section 3.1. For example, the time-declining target would yield extremely high occupancy targets in short intervals and, consequently, cannot completely avoid oversaturation caused by mass passenger arrivals. Additionally, the parameters are set for undersaturated transit systems, of which the undersaturation cannot be guaranteed in the current model with uncertain passenger arrivals. More discussions are made in Section 3.1.

To address the aforementioned issues, this paper extends the original D-O model to a more general form, called the revised D-O model, of which the parameters are to be wisely chosen according to the demand distributions. We examine the performance of the revised D-O model under various circumstances, where the demand follows distributions with low to high variances. We also propose an alternative simple model with a linear target function for comparison with the original and revised D-O models. For illustration, we focus the study on shuttle transit, of which the single core decision is the timing of vehicle dispatches. However, the proposed models can also be applied to other non-reservation-based transit, such as fixed- and flexible-route transit (Kim, Levy, and Schonfeld 2019; Nourbakhsh and

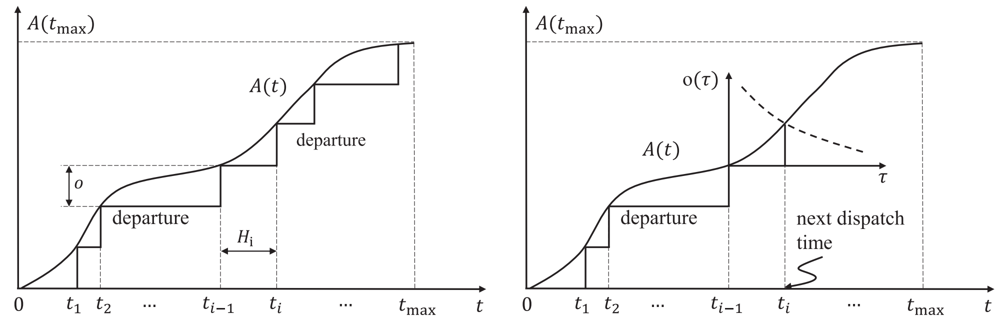
Ouyang 2012; P. W. Chen and Nie 2017) and on-demand ride-pooling services (Jung, Jayakrishnan, and Park 2016; Lotfi, Abdelghany, and Hashemi 2019; W. Fan, Gu, and Xu 2024; Xi et al. 2023), as discussed in the last part of this paper. For reservation-based transit, such as dial-a-ride, school buses, and subscription buses, the demand can be considered known or with little uncertainty, allowing for the design and publication of schedules in advance. Comparisons between adaptive dispatch strategies and traditional schedule-based ones would help determine the most suitable operation and its associated conditions.

Specifically, W. Fan and Wang (2024) proposed an integrated optimisation model that jointly considers adaptive dispatching and zoning using a soft-target strategy, aiming to provide feeder services for *round-trip* demand between transport depot and suburbs. Passengers are dispersed across suburban areas. The vehicle pick up outbound passengers and drop off inbound passengers within its service zone of the suburbs. In contrast to the above work, this paper focuses on shuttle services for *one-way* demand at a transport depot. These passengers arrive randomly at the depot in concentrated patterns to return to their destinations. In the research context of shuttle services focussed on dispatching strategies, the impact of soft-target strategies on system performance can be comprehensively demonstrated. This avoids interference from other decisions (such as service zone). Moreover, this setting facilitates a fair comparison between the proposed models and the models in existing literature.

1.2. Our contributions

We focus on transit services with low average demand levels but significant variations. These circumstances are frequently observed in suburbs (e.g. peripheral regions) and during non-peak times of day. Thus, our objective is to prevent unexpected oversaturation and cost-inefficient undersaturation, as well as to avoid excessive waiting delays. The contributions of this study are summarised as follows:

(1) We optimise the revised D-O and the linear soft-target models to find the best parameter values based on the known statistical distributions of passenger arrivals. Compared with the original D-O model, the revised models offer an inherent advantage in dynamic optimisation in response to changes in demand distributions.
(2) We conduct extensive simulations and numerical experiments to validate the accuracy and effectiveness of the proposed models. The results indicate that the proposed two strategies outperform their counterparts (e.g. the hard-target and the original D-O models) under demand distributions with large variances and low means. The relative performance of the proposed models is influenced by passengers' values of time (VOT), with those of moderate VOTs benefiting the most from the proposed strategies.
(3) Sensitivity analysis reveals that although the proposed models may require extra vehicles, they maintain superiority under conditions of large variances and low means or moderate VOTs due to better user experience.
(4) We demonstrate the application of the proposed models through a case study of an airport shuttle line in Beijing, China. We evaluate the model's performance and propose two mitigation strategies under stringent fleet size constraints imposed on operators. Furthermore, considering potential measurement errors in real-world



(a) Hard-target dispatch strategy (b) Soft-target dispatch strategy

Figure 1. Cumulative arrivals and departures under hard- and soft-target dispatch strategies (Daganzo and Ouyang 2019). (a) Hard-target dispatch strategy and (b) Soft-target dispatch strategy.

demand distribution data, we provide the model's performance boundaries under varying demand means and variances.

It is noted that recent research has investigated online RL approaches to adaptive vehicle scheduling (Ai et al. 2022; Chow, Li, and Ying 2024). Nevertheless, the soft-target models yield closed-form analytical solutions under known parameters of the demand distribution, enabling real-time computation without necessitating iterative updates or extensive offline training. Conversely, RL-based methods typically rely on substantial historical data for initial learning and continuous environmental feedback for online updates, thereby incurring substantial computational overhead during operation. In transit systems with limited sensing infrastructure, these data may not be available or are costly to obtain. It is also noted that the RL-based method may converge to the local optimum, while the proposed models in this paper can achieve the global optimum, which we will describe later.

The remainder of this paper is organised as follows. Section 2 describes the vehicle dispatch problem of the study. Section 3 proposes new models of soft-target dispatch strategies after reviewing the existing ones. Section 4 examines the accuracy of the proposed models through simulations, followed by comparisons with the counterparts in Section 5. Section 6 presents a discussion on the pros and cons, as well as the applicable scenarios, of all soft-target models. The last section concludes the paper and briefly discusses future research topics.

2. Problem description

We consider a shuttle transit that serves passengers travelling between an origin-destination pair for a time horizon $[0, t_{max}]$. The actual demand can be described by a cumulative arrival curve $A(t) \equiv \int_0^t \lambda(u)\,\mathrm{d}u, t \in [0, t_{max}]$, where $\lambda(u)$ is the passenger arrival rate at time u, as shown in Figure 1(a,b).

In Figure 1(a), the hard-target strategy dispatches buses when the occupancy target, denoted o, is reached, i.e. $o = A(t_i) - A(t_{i-1}) = \int_{t_{i-1}}^{t_i} \lambda(u)\,\mathrm{d}u$, where t_i, t_{i-1} mark the adjacent ith and $(i-1)$th dispatch times and $H_i = t_i - t_{i-1}$ defines the ith headway. As seen, the length of headway H_i is not bounded under the hard-target strategy if $\lambda(u)$ were low. Consequently, the arrived passengers may have to endure an overly lengthy wait.

A remedy can be provided, as shown in Figure 1(b), where a soft occupancy target is introduced as a decreasing function of the elapsed time τ since the last dispatch, expressed as $o(\tau)$ (Daganzo and Ouyang 2019). Then, the intersection point of the $o(\tau)$ and $A(t)$ curves marks the dispatch time t_i and headway H_i. In other words, $o(H_i) = A(t_{i-1} + H_i) - A(t_{i-1}) = \int_{t_{i-1}}^{t_{i-1}+H_i} \lambda(u)du$ holds at any dispatch time $t_i = t_{i-1} + H_i$, $\forall\ i$. Since $o(\tau)$ is a decreasing function by definition, it will intersect with $A(t)$ within a finite headway, i.e. $o(H_i) \geq 0$ implying $H_i \leq \tau^{\max}$, where $\tau^{\max}$ satisfies $o(\tau^{\max}) = 0$. As a result, the length of headway H_i is bounded under the soft-target strategy and long waits are effectively prohibited.

The main ingredient in the soft-target strategy is the time-declining function $o(\tau)$ that is to be studied in this paper. It is evident how fast the target function declines affects transit operator's costs and passengers' waiting time. A balanced trade-off must be made between the two aspects. To facilitate the modelling, we adopt the following assumptions:

(1) Following previous studies (e.g. S. K. Chang and Schonfeld 1993; Z. Chen, Li, and Zhou 2020), this paper considers a sufficient fleet of vehicles at the depot. The fleet size problem can be evaluated afterward (Ceder and Stern 1981).
(2) Passengers randomly arrive at the origin station and follow the known stochastic processes, which may be time-dependent and change among different periods of the day (Hong and Chen 2017; J. Yin et al. 2016). This knowledge can be obtained by statistical analysis of historical data or by prediction.
(3) The arrived passengers do not exit the transit system while waiting to board (Patel and Gor 2025). According to previous research (Ceder 2016), the main reason for passengers to leave the system is that they fail to board due to excessively long waiting times. Avoiding long waiting times is the core advantage of the soft-target models, which generate acceptable waiting time for passengers. This is verified in the following text.

3. Methodologies

This section formulates the vehicle dispatch problem using mathematical models. The notations used in this paper are summarised in Table 1 for the reader's convenience. Section 3.1 first reviews the hard-target and the original D-O models. Section 3.2 and 3.3 develop two new soft-target strategies, namely the revised D-O model and the linear model, of which two special cases are given in Section 3.4.

3.1. Review of two dispatch models

3.1.1. Hard-target strategy

The hard-target strategy optimises the vehicle occupancy target o by minimising the generalised system cost as the weighted sum of passengers' waiting time and the operator's operational cost (Daganzo and Ouyang 2019). They are formulated as follows.

Table 1. Nomenclature.

Notation	Description	Unit
Random Variables		
λ	Stochastic process of passenger arrival rates	passenger/hour
λ_i	Average passenge arrival rates in time interval between $(i-1)$th and ith dispatches	passenger/hour
$A(t_{\max})$	Cumulative number of passenger arrival	passenger
$\bar{\lambda}$	Average arrival rate in time horizon	passenger/hour
Decision Variables		
o	Vehicle occupancy target	passenger
γ, α	Parameters in revised D-O model	passenger · hour$^{\alpha}$,/
θ	Coefficient of linear target model	passenger/hour
Parameters		
t	Time index	hour
$t_{\max}$	Length of the study period	hour
t_i	Time of ith dispatch	hour
τ	Elapsed time since last dispatch	hour
$o(\tau)$	Occupancy target function	passenger
κ	Coefficient of variation	/
c_f	Unit operational cost	\$/dispatch
β	Passengers' value of time (VOT)	\$/(hour·passenger)
C	Vehicle capacity	passenger/vehicle
Z	Expected generalised system cost	\$
Λ_i	Deterministic passenger arrival rate of the ith dispatch	passenger/hour
H_i	Headway of the ith dispatch	hour
p	Index of time period	/
μ_p, σ_p	Mean & standard deviation of demand distribution	passenger/hour

First, passengers' waiting time per unit time can be computed by

$$\frac{1}{t_{\max}}\beta\sum_i o\frac{1}{2}H_i = \frac{1}{2}\beta o, \tag{1}$$

where β denotes passengers' average value of time (VOT). Equation (1) implies the transit system is undersaturated, and thus, the average waiting time of passengers in any headway interval is $H_i/2$, $\forall\, i$.

Second, the operational cost per unit of time can be given by

$$\frac{c_f}{t_{\max}}\frac{A(t_{\max})}{o} = c_f\frac{\bar{\lambda}}{o}, \tag{2}$$

where c_f is the unit operational cost per dispatch, which includes all operational costs (e.g. the driver's wages, vehicle fuel and vehicle purchase,[1] etc); $\frac{A(t_{\max})}{o}$ returns the total number of dispatches; and $\bar{\lambda} \equiv A(t_{\max})/t_{\max}$ represents the average arrival rate during the study period. Note that $\bar{\lambda}$ is a random variable since $A(t_{\max})$ is.

Then, an optimisation problem can be constructed to minimise the expected generalised system cost, denoted Z,

$$\underset{o}{minimise}\, Z = \frac{1}{2}\beta o + c_f\frac{\mathrm{E}(\bar{\lambda})}{o}, \quad \text{subject to} : o \le C, \tag{3}$$

where $\mathrm{E}(\cdot)$ returns the expectation of the argument; and C is the vehicle capacity.

Solving the above problem yields the optimal target o^* as given by

$$o^* = \min\left\{\sqrt{\frac{2c_f \mathrm{E}(\bar{\lambda})}{\beta}}, C\right\}. \tag{4}$$

Note that the above process can be applied to scenarios with multiple periods (e.g. morning and evening peaks and off-peak hours) indexed $p = 1, 2, \ldots$ Accordingly, Equation (4) can update the hard target o_p^* according to the time-dependent demand $\bar{\lambda}_p$ in each period p.

3.1.2. Original D-O model

Daganzo and Ouyang (2019) propose choosing a soft target model $o(\tau_i)$ to satisfy for any dispatch i that the elapsed time (τ_i) equals the optimal headway (H_i^*), which minimises the generalised system cost of *unconstrained* transit services with *deterministic* demand (of which passengers' arrival rate is denoted Λ_i), i.e.

$$\tau_i = H_i^* = \mathrm{argmin}_{H_i} Z_i = \beta\Lambda_i \frac{H_i}{2} + c_f \frac{1}{H_i}, \tag{5}$$

where $\Lambda_i = \frac{o(\tau_i)}{\tau_i}, \forall i \in \{1, 2, \ldots\}$.

From (5), one can obtain the condition $\tau_i = H_i^* = \sqrt{\frac{2c_f}{\beta\Lambda_i}} = \sqrt{\frac{2c_f\tau_i}{\beta o(\tau_i)}}$, $\forall$ i, which implies a soft-target model as follows

$$\tilde{o}(\tau) = \frac{2c_f}{\beta\tau}. \tag{6}$$

This result is *unrelated* to demand, and thus, cannot guarantee the transit system being undersaturated, i.e. $\Lambda_i H_i^* \le C$, $\forall$ i. To see it, consider a small time interval, for example, $\tau \to 0$, the target $o(\tau) \gg C$ holds; in this case, the excessive passengers $(o(\tau) > \Lambda_i\tau > C)$ would not trigger a dispatch. Generally, any crowds $(\Lambda_i\tau > C)$ arriving within $\tau = [0, \frac{2c_f}{\beta C})$ will lead to oversaturation, meaning some passengers must wait for multiple vehicle arrivals before boarding.

In practice, however, this oversaturation issue in Equation (6) can be simply amended by adding a boundary constraint, as in

$$o(\tau) = \min\left\{\tilde{o}(\tau), C\right\}, \tag{7}$$

such that vehicles will be dispatched when either the occupancy target or the vehicle capacity is reached.

Note that Equation (7) can also be obtained from Equation (4) by replacing the hard target o^* with the soft target $o(\tau)$ and the average arrival rate over the entire study period $\mathrm{E}(\bar{\lambda})$ by the average within any interval $o(\tau)/\tau$.

Nonetheless, we argue that the above result ($\tilde{o}(\tau)$ in Equation (7)), obtained from the optimisation for unconstrained and deterministic systems, would not be optimal under situations with high demand uncertainty. Although the amendment in Equation (7) resolves the oversaturation issue, it would inevitably compromise the optimisation in Equation (5). Therefore, we propose a revised D-O model as described in the next section and will demonstrate its advantage over the original model in the following text.

3.2. Revised D-O model

We propose the revised D-O model in a general form of

$$o(\tau) = \frac{\gamma}{\tau^{\alpha}}, \tag{8}$$

where γ and α are two parameters to be determined in the following optimisation problem.

The revised D-O model satisfies for any dispatch headway H_i, $\forall\, i = 1, 2, \ldots$,

$$o\,(\tau = H_i) = \frac{\gamma}{H_i^{\alpha}} = \lambda_i H_i, \tag{9}$$

where λ_i is a random variable (with certain distributions, e.g. normal distributions), representing the average passenger arrival rates in the time interval between $(i-1)$th and ith dispatches. For clarity, subscript i is not added to parameters γ and α.

Equation (9) implies that the dispatch headways satisfy $H_i = (\frac{\gamma}{\lambda_i})^{\frac{1}{\alpha+1}}$. Accordingly, we can formulate an optimal design problem to minimise the expected generalised system cost Z_i for any dispatch $i = 1, 2, \ldots$ with vehicle capacity constraint, as given below

$$\begin{aligned} \underset{\gamma,\alpha}{minimise}\, Z_i(\gamma,\alpha) &= \mathrm{E}\left[\beta\lambda_i \frac{1}{2} H_i + c_f \frac{1}{H_i}\right] \\ &= \mathrm{E}\left[\beta\lambda_i^{\frac{\alpha}{\alpha+1}} \frac{1}{2}\gamma^{\frac{1}{\alpha+1}} + c_f \lambda_i^{\frac{1}{\alpha+1}} \left(\frac{1}{\gamma}\right)^{\frac{1}{\alpha+1}}\right], \end{aligned} \tag{10a}$$

subject to:

$$\begin{aligned} &\mathrm{E}\,(\lambda_i H_i) + 2\mathrm{STD}\,(\lambda_i H_i) \\ &\quad = \gamma^{\frac{1}{\alpha+1}} \left[\mathrm{E}\left(\lambda_i^{\frac{\alpha}{\alpha+1}}\right) + 2\sqrt{\mathrm{E}\left(\lambda_i^{\frac{2\alpha}{\alpha+1}}\right) - \mathrm{E}^2\left(\lambda_i^{\frac{\alpha}{\alpha+1}}\right)}\right] \le C, \end{aligned} \tag{10b}$$

$$\gamma > 0, \quad \alpha > 0, \tag{10c}$$

where the first and second terms in the bracket of Equation (10a) are the passengers' waiting time cost and the operator's operational cost, respectively. In Equation (10b), $\mathrm{STD}(\cdot)$ returns the standard deviation (STD) of the argument; and the inequality constraint ensures that the transit systems are most likely undersaturated. Note that the expectation $\mathrm{E}(\lambda_i^{\frac{\alpha}{\alpha+1}})$ can be computed via numerical integration once the probability distribution of λ_i is specified. Specifically, let $f(\lambda_i)$ denotes the known probability density function of λ_i. Then the expectation is given by $\mathrm{E}(\lambda_i^{\frac{\alpha}{\alpha+1}}) = \int_0^{\infty} \lambda_i^{\frac{\alpha}{\alpha+1}} f(\lambda_i)\, \mathrm{d}\lambda_i$. The last constraint stipulates the valid ranges for two parameters in the revised D-O model.

The optimisation of Equation (10a) will be performed for every dispatch, implying that the revised D-O model dynamically updates its parameters γ and α based on changes in demand distributions. In contrast, the original D-O model (Equation (6)) has fixed parameter values regardless of demand changes. It is expected that this advantage of the revised D-O model will lead to superior performance, as will be demonstrated in numerical studies.

To solve Equation (10a), we first treat decision variable α as known and then find the optimal γ^*. We let $x = \gamma^{\frac{1}{\alpha+1}}$, and then the optimisation problem (10a) is re-written as:

$$\underset{x}{minimise}\, Z_i(x\,|\,\alpha) = \left[\beta \mathrm{E}\left(\lambda_i^{\frac{\alpha}{\alpha+1}}\right)\frac{1}{2}x + c_f \mathrm{E}\left(\lambda_i^{\frac{1}{\alpha+1}}\right)\frac{1}{x}\right], \tag{11a}$$

$$\text{s.t.} \begin{cases} x\left[\mathrm{E}\left(\lambda_i^{\frac{\alpha}{\alpha+1}}\right) + 2\sqrt{\mathrm{E}\left(\lambda_i^{\frac{2\alpha}{\alpha+1}}\right) - \mathrm{E}^2\left(\lambda_i^{\frac{\alpha}{\alpha+1}}\right)}\right] \le C \\ x > 0 \end{cases}. \tag{11b}$$

It can be verified that the above subproblem (11a) is a convex programming concerning decision variable x because $\frac{\partial^2 Z_i(x|\alpha)}{\partial x^2} = 2c_f \mathrm{E}(\lambda_i^{\frac{1}{\alpha+1}})x^{-3} > 0$ for $\forall\ x > 0$. Further, the problem (10a) is a convex one-dimensional subproblem concerning decision variable x combined with a one-dimensional search over α. Thus, solving Equation (11a) yields the optimal x^* and subsequently the optimal γ^* as

$$\gamma^*(\alpha) = \min\left\{\left[\frac{2c_f \mathrm{E}\left(\lambda_i^{\frac{1}{\alpha+1}}\right)}{\beta \mathrm{E}\left(\lambda_i^{\frac{\alpha}{\alpha+1}}\right)}\right]^{\frac{\alpha+1}{2}}, \left(\frac{C}{U}\right)^{\alpha+1}\right\}, \tag{12}$$

where $U \equiv \mathrm{E}(\lambda_i^{\frac{\alpha}{\alpha+1}}) + 2\sqrt{\mathrm{E}(\lambda_i^{\frac{2\alpha}{\alpha+1}}) - \mathrm{E}^2(\lambda_i^{\frac{\alpha}{\alpha+1}})}$.

Next, we suggest employing a one-dimensional linear search algorithm to find the optimal α^* after substituting Equation (12) back to Equation (10a). Note that to guarantee no oversaturation in practice, a similar amendment should be made to the revised D-O model as follows:

$$o^*(\tau) = \min\left\{\frac{\gamma^*}{\tau^{\alpha^*}}, C\right\}. \tag{13}$$

3.3. Linear model

We note in both the original and revised D-O models that the boundary constraint may be reached in practice, i.e. $\frac{2c_f}{\beta\tau} \ge C$ and $\frac{\gamma^*}{\tau^{\alpha^*}} \ge C$, implying $o(\tau) = C$ by Equations (7) and (13), for small τ's. This would potentially undermine the benefit brought by the optimizations. In light of this, we propose an alternative model with a linear target function as follows

$$o(\tau) = C - \theta\tau, \quad \text{where, } \theta \ge 0. \tag{14}$$

Clearly, $o(\tau)$ should not be negative or equal to zero; otherwise, vehicles might be dispatched before any passengers arrive. Thus, we specify $\tau \in [0, (C-1)/\theta]$ in Equation (14) and any larger τ's are truncated as $(C-1)/\theta$. As a result, $o(\tau) \in [1, C]$ is always satisfied. Note that the original and revised D-O models do not need to truncate τ since their targets will not reach zero for $\tau \ge 0$ and no vehicle would be dispatched unless at least one passenger had arrived.

Parameter θ in Equation (14) can be optimised in a similar way as the above section. The optimal design problem for dispatch *i* with the proposed linear model can be formulated as

$$\underset{\theta}{minimise}\, Z_i(\theta) = \mathrm{E}\left[\beta\lambda_i\frac{1}{2}\frac{C}{\lambda_i+\theta} + c_f\frac{\lambda_i+\theta}{C}\right], \quad \text{s.t., } \theta \geq 0. \tag{15}$$

Since only one decision variable θ exists, the one-dimensional linear search approach can also be applied to find the optimum solution θ^*. Similar to the revised D-O model, the linear model will also be dynamically updated concerning the changes in the mean of λ_i.

3.4. Two special cases in scenarios with deterministic demand

3.4.1. Special case of revised D-O model

When passengers' arrival rates are deterministic, denoted Λ_i, $\forall\, i = 1, 2, \ldots$, the optimal solution of γ by Equation (12) can be reformulated as below:

$$\gamma^*(\alpha) = \min\left\{\left(\frac{2c_f}{\beta}\right)^{\frac{\alpha+1}{2}} \Lambda_i^{\frac{1-\alpha}{2}}, \frac{C^{\alpha+1}}{\Lambda_i^{\alpha}}\right\}. \tag{16}$$

The dispatch headway satisfies

$$\frac{\gamma^*(\alpha)}{H_i^{\alpha}} = \Lambda_i H_i, \tag{17}$$

which implies

$$H_i = \left[\frac{\gamma^*(\alpha)}{\Lambda_i}\right]^{\frac{1}{\alpha+1}} = \min\left\{\sqrt{\frac{2c_f}{\beta\Lambda_i}}, \frac{C}{\Lambda_i}\right\}. \tag{18}$$

The above result is independent of parameters γ and α. In other words, there is no need to introduce the soft target concept when the demand is certain.

3.4.2. Special case of linear model

With deterministic demand, the optimal solution of θ becomes

$$\theta^* = \sqrt{\frac{\beta\Lambda_i}{2c_f}}C - \Lambda_i. \tag{19}$$

The dispatch headway satisfies

$$C - \theta^* H_i = \Lambda_i H_i, \quad \forall\, \theta^* \geq 0, \tag{20}$$

implying

$$H_i = \frac{C}{\Lambda_i + \theta^*} = \min\left\{\sqrt{\frac{2c_f}{\beta\Lambda_i}}, \frac{C}{\Lambda_i}\right\}, \tag{21}$$

which is also independent of the soft-target model parameter.

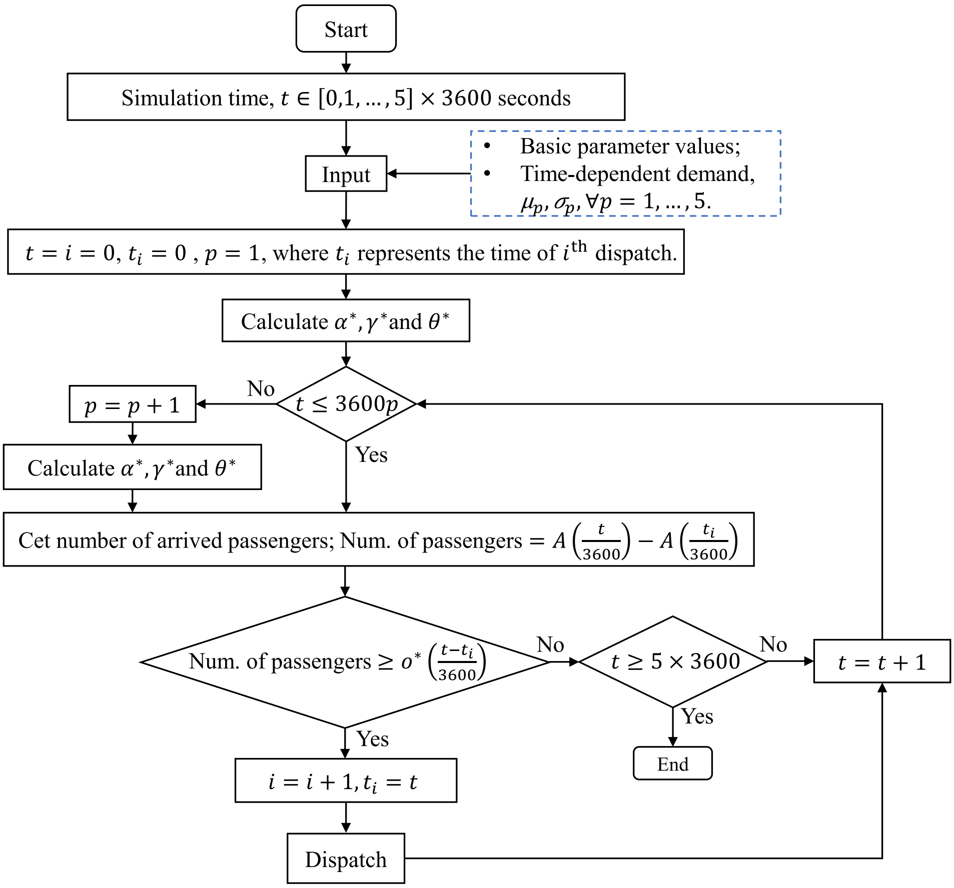
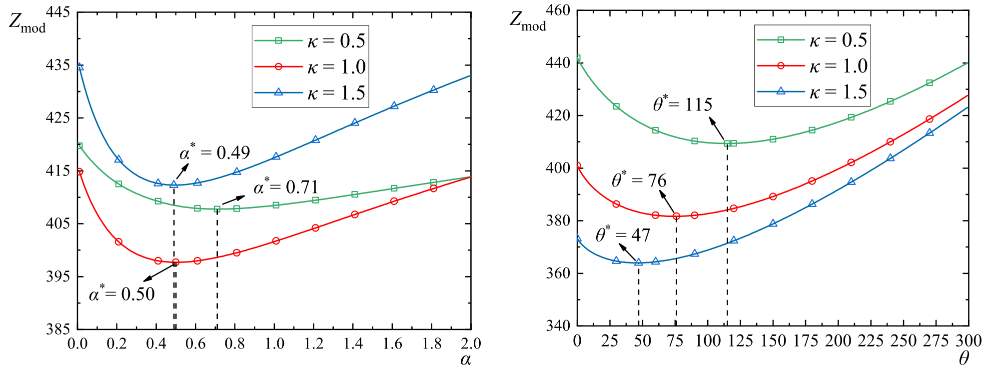


Figure 2. Flow chart of the simulation process.

4. Simulation tests

We simulate to verify the accuracy of the proposed models. The simulation flow chart is shown in Figure 2. The simulation is from 07:00 to 12:00, denoted as $t \in [0, 5 \times 3600]$ with the timestamp of 1 second, which is uniformly divided into five periods, $p = 1, \dots, 5$, for illustration. We set $\beta = 20$ [\$/(hour·passenger)] for a high-wage city (e.g. Barcelona, Spain) and adopt $C = 30$ [passengers/vehicle] for regular buses with $c_f = 15$ [\$ per dispatch] (Kim, Levy, and Schonfeld 2019; W. Fan, Mei, and Gu 2018).

Following previous studies (Gong et al. 2016; Hong and Chen 2017), we consider two demand scenarios where passengers' arrivals follow normal and gamma distributions with time-dependent means (μ_p) and STDs (σ_p). We set the following default values to reflect unimodal demand profiles in a typical morning peak, $[\mu_1, \mu_2, \mu_3, \mu_4, \mu_5] = [100, 200, 300, 200, 100]$ [passengers/hour]. The STDs are $\sigma_p = \kappa\mu_p$, $\forall\ p$, where $\kappa \in \{0.5, 1.0, 1.5\}$ is the coefficient of variation (CV). The three CV values represent demand scenarios with low, medium, and large variations. Note that gamma distributions reduce to exponential distributions when $\kappa = 1.0$ and $\sigma_p = \mu_p$.



(a) Changes in system cost (Z_{mod}) concerning α in revised D-O model

(b) Changes in system cost (Z_{mod}) concerning θ in linear model

Figure 3. Search for optimal parameter values in the proposed models under normal distribution with mean of 300 passengers/h. (a) Changes in system cost (Z_{mod}) concerning α in revised D-O model and (b) Changes in system cost (Z_{mod}) concerning θ in linear model.

Table 2. Differences in generalised system cost between the simulations and the models.

Demand		Revised D-O model		Linear model	
mean	STD	normal	gamma	normal	gamma
μ_p	$0.5\mu_p$	−0.82%	1.78%	1.09%	1.70%
	$1.0\mu_p$	−0.27%	1.79%	−1.41%	0.84%
	$1.5\mu_p$	−1.95%	−0.33%	−1.58%	0.45%
$0.5\mu_p$	$1.0\mu_p$	0.58%	0.45%	−1.39%	0.97%
$1.0\mu_p$		−0.27%	1.79%	−1.41%	0.84%
$1.5\mu_p$		0.63%	1.34%	0.93%	0.64%

Given the above demand profiles, the proposed models (i.e. Equations (13) and (14)) take $\mathrm{E}(\lambda_i) = \mu_p$ and $\mathrm{STD}(\lambda_i) = \sigma_p$ for each dispatch decision, if the last dispatch time t_{i-1} is within pth period. Thus, the parameter values of the proposed models will be optimised and updated for every period $p = 1, \ldots, 5$. (Note that out of practical interest, if demand distributions vary more significantly in time, additional periods can be divided to dynamically update the proposed models.) Figure 3(a,b) illustrate the process of finding the optimal α^* and θ^*, under the demand scenario of the normal distribution for example.

We conduct 1000 simulations for each demand scenario and compare the averaged results of the simulations with those of the proposed models. Table 2 summarises the differences in the generalised system cost, measured by $\frac{Z_{\text{mod}} - Z_{\text{sim}}}{Z_{\text{mod}}} \times 100\%$, where subscripts 'mod' and 'sim' stand for the models and simulations, respectively. Rows three to five show results for demand distributions with the same mean but different STDs. Rows six to eight display results for demand distributions with different means but the same STD. As seen, our models are sufficiently accurate as the absolute differences are consistently less than 2% in all scenarios.

Table 3. Search for optimal parameter values with different step under normal distribution with mean of 300 passengers/h.

	α in revised D-O model						θ in linear model			
	step size $= 0.01$			step size $= 0.001$			step size $= 1$		step size $= 0.1$	
STD	α^*	γ^*	time	α^*	γ^*	time	θ^*	time	θ^*	time
150 ($\kappa = 0.5$)	0.71	3.01	27	0.712	2.994	292	115	17	115.1	163
300 ($\kappa = 1.0$)	0.50	4.08	20	0.503	4.039	210	76	12	75.8	123
450 ($\kappa = 1.5$)	0.49	3.69	23	0.495	3.629	213	47	9	47.0	85

*Time: the search duration for obtaining optimal solution (unit: seconds).

4.1. The accuracy of solution

The globally optimal solution is determined through linear search using step size of 0.01 for $\alpha \in (0, 1]$. For parameter θ of the linear model, considering that the unit of θ is [passenger/hour], we employed a step size of 1 to search for the optimal value within range of 0 to 150. To verify whether this level of precision is sufficient, the search step sizes are further reduced to 0.001 for α and 0.1 for γ, respectively, and the results are presented in Table 3.

The results indicate that an excessively small search step size leads to substantial increase in computational time. The resulting highly precise parameter values yield changes in departure times within a few seconds, which hold meaningless for real-world operations. For example, in $k = 1$ scenario, considering a scenario where the vehicle departs upon reaching a passenger count of 15, the departure time of linear model under the current precision is 710.5 seconds, whereas that obtained with a step size of 0.1 is 712.4 seconds. For revised D-O model, the departure time under the current precision is 266.3 seconds, whereas that obtained with a step size of 0.001 is 265.1 seconds. Thus, the solution accuracy is sufficient.

4.2. Headway stability for linear model

For linear model, we introduce a truncation at $\tau = (C - 1)/\theta$ for any larger τ in Equation (14). In fact, the cumulative arrival curve $A(t)$ of the passengers is an increasing step function. This abrupt truncation does not cause extreme instability in the headway because the minimum unit for arriving passengers is 1 (W. Fan et al. 2025). That is, the declining form of the function when the target value $o(\tau) \leq 1$ does not affect the departure time. Figure 4 shows the vehicle departure process of the linear model under two different distributions. It is noted that the headway will change dynamically as passengers arrive. This dynamic change in the headway is to adapt to demand fluctuations rather than being caused by the setting of $\tau = (C - 1)/\theta$.

5. Numerical experiments

In this section, we will examine the performances of the proposed models compared to their counterparts (i.e. the hard-target and the original D-O models) under different demand distributions. It is noteworthy that the time-declining function $o(\tau) = \frac{2c_f}{\beta\tau}$ in the original D-O model is independent of demand in each time period; it varies solely with parameters c_f and β. This inherent form is determined by the model framework of Daganzo and Ouyang (2019). We maintain consistency with the original D-O model's specification in

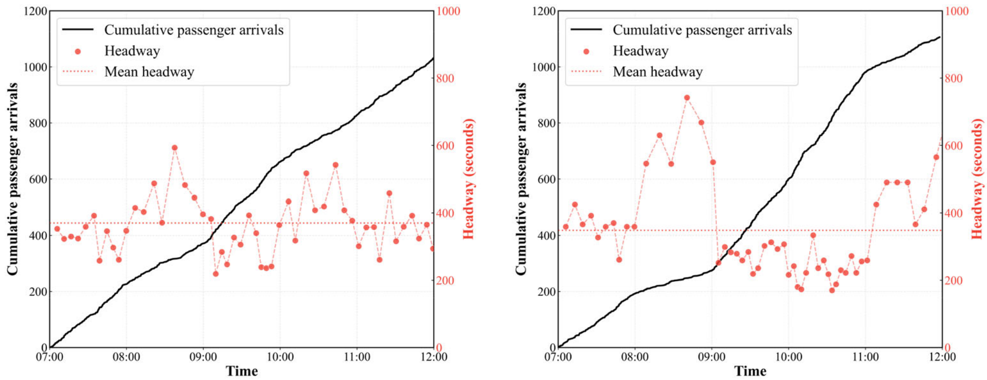


Figure 4. An operational process example for linear model under normal distribution (left) and gamma (right) distribution with default means μ_p and STDs $\sigma_p = \mu_p$.

Table 4. CVs of headways under soft- and hard-target models.

Demand		Hard-target model		Original D-O model		Revised D-O model		Linear model	
mean	STD	normal	gamma	normal	gamma	normal	gamma	normal	gamma
μ_p	$0.5\mu_p$	0.30	0.25	0.12	0.14	0.10	0.14	0.14	0.16
	$1.0\mu_p$	0.60	0.60	0.22	0.30	0.48	0.34	0.27	0.32
	$1.5\mu_p$	2.56	1.10	0.83	0.50	1.00	0.64	0.41	0.51
$0.5\mu_p$	$1.0\mu_p$	2.52	1.97	0.62	0.81	0.66	0.86	0.38	0.54
$1.0\mu_p$		0.60	0.60	0.22	0.30	0.48	0.34	0.27	0.32
$1.5\mu_p$		0.38	0.35	0.18	0.19	0.24	0.23	0.23	0.23

this section. Therefore, in scenarios with only demand variation, the time-declining functional form of the original D-O model remains fixed, while that of the other models is updated dynamically according to the demand level. In the sensitivity analysis experiments regarding VOTs, all soft-target models dynamically update their time-declining functions.

Section 5.1 analyzes the CV in headway, followed by the generalised system cost comparisons in Section 5.2. Section 5.3 assesses the likelihood of the soft target taking the value of vehicle capacity, i.e. $o^*(\tau) = C$, which indicates soft-target models reduce to hard-target ones. Sections 5.4 and 5.5 present the impact of passengers' VOTs and fleet size cost, respectively. Finally, a real case study in Section 5.6 demonstrates the application of the proposed models to an airport shuttle line in Beijing (China).

5.1. *Coefficient of variation in headway*

Since passengers' waiting time is proportional to headway, we adopt CV of headway, defined by $\frac{\text{STD}(H_i)}{\text{E}(H_i)}$, to verify the effectiveness of soft-target models in preventing excessively long waiting times for passengers.

Tables 4 and 5 summarise the comparisons between the hard- and soft-target models, i.e. the original and revised D-O models and the linear model. Note that for a fair comparison, the hard-target model is also updated for every period. As observed in Table 4, the soft-target models significantly reduce the CVs in comparison to the hard-target model.

Table 5. Means of headways under soft- and hard-target models [seconds].

Demand		Hard-target model		Original D-O model		Revised D-O model		Linear model	
mean	STD	normal	gamma	normal	gamma	normal	gamma	normal	gamma
μ_p	$0.5\mu_p$	312	316	312	316	292	316	310	314
	$1.0\mu_p$	304	370	290	322	268	312	286	316
	$1.5\mu_p$	332	516	274	348	246	298	264	328
$0.5\mu_p$	$1.0\mu_p$	492	1090	420	608	350	510	364	550
$1.0\mu_p$		304	370	290	322	268	312	286	316
$1.5\mu_p$		264	276	252	254	224	250	246	250

Table 6. Expected number of dispatches under soft- and hard-target models.

Demand		Hard-target model		Original D-O model		Revised D-O model		Linear model	
mean	STD	normal	gamma	normal	gamma	normal	gamma	normal	gamma
μ_p	$0.5\mu_p$	55	53	52	52	57	51	51	52
	$1.0\mu_p$	59	54	49	48	54	49	48	49
	$1.5\mu_p$	66	54	50	45	60	51	48	45
$0.5\mu_p$	$1.0\mu_p$	53	39	37	27	38	27	33	25
$1.0\mu_p$		59	66	49	64	54	65	48	65
$1.5\mu_p$		66	54	63	48	70	49	62	49

A closer observation finds that the revised D-O and linear models may have CVs slightly larger than the original D-O model under certain scenarios. However, Table 5 shows that the means of headways in the proposed two models are always less than or equal to that of the original D-O model. In other words, an average passenger benefits from the least waiting time in the proposed models. The results speak to the effectiveness of the proposed soft-target models in avoiding excessively long waiting times.

We also notice that the soft-target models may increase the number of dispatches under some scenarios, as shown in Table 6. As a result, the operational cost may be increased. Thus, we are interested in finding how the generalised system cost, which combines passengers' waiting time and the operator's cost, changes in the proposed models as compared to their counterparts.

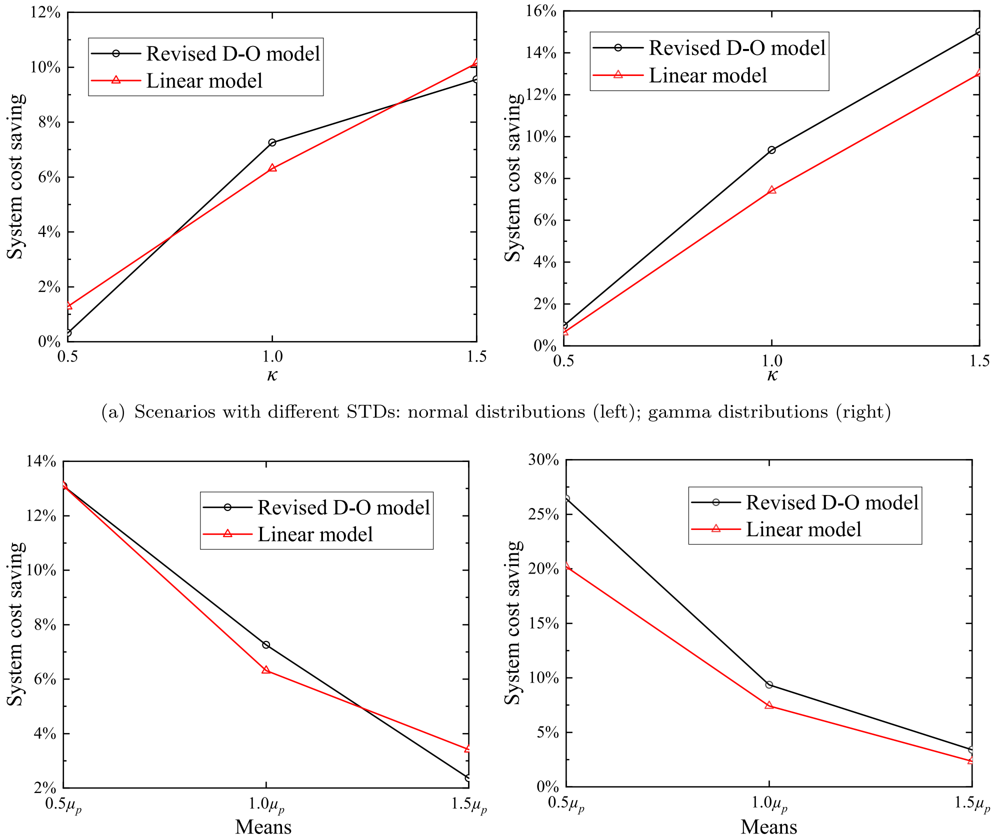
5.2. Generalised system cost saving

We define the system cost saving by $\frac{Z_{\text{counterpart}}-Z_{\text{proposed}}}{Z_{\text{counterpart}}} \times 100\%$, where subscripts 'proposed' and 'counterpart' denote the proposed model and its counterpart, respectively.

Figures 5 and 6 present the comparison results with the hard-target and original D-O models, respectively. As seen in Figure 5, the proposed two models dominate over the hard-target model over all scenarios with system cost savings of up to 26%. The superiority manifests remarkably under demand distributions with larger STDs (see Figure 5(a)) and lower means (Figure 5(b)).

Figure 6 indicates that our models consistently outperform the original D-O model across all scenarios, due to the time-dependent optimisation. The advantage of the proposed models is confirmed again under demand distributions with larger STDs and lower means. The cost saving against the original D-O model can reach up to 18%.

The relative performances of the two new soft-target models are close. The revised D-O model generally performs better under gamma (and exponential) distributions, while the



(a) Scenarios with different STDs: normal distributions (left); gamma distributions (right)

(b) Scenarios with different means: normal distributions (left); gamma distributions (right)

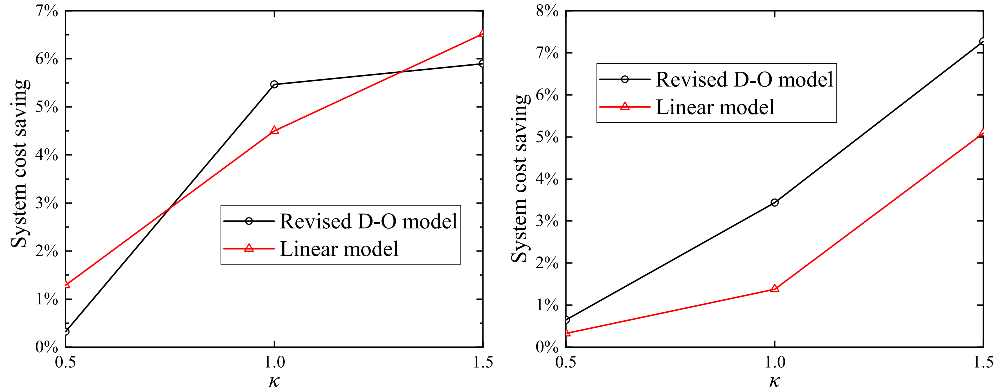
Figure 5. System cost saving of two soft-target models compared with the hard-target model. (a) Scenarios with different STDs: normal distributions (left); gamma distributions (right) and (b) Scenarios with different means: normal distributions (left); gamma distributions (right).

linear model shows an advantage under normal distributions. The proposed models produce less benefit as the mean of demand distributions grows and would eventually reduce to the hard-target model with $o^*(\tau) = C$. (Combining the results in Figures 5 and 6 implies that the original D-O model performs slightly better than the hard-target model.)

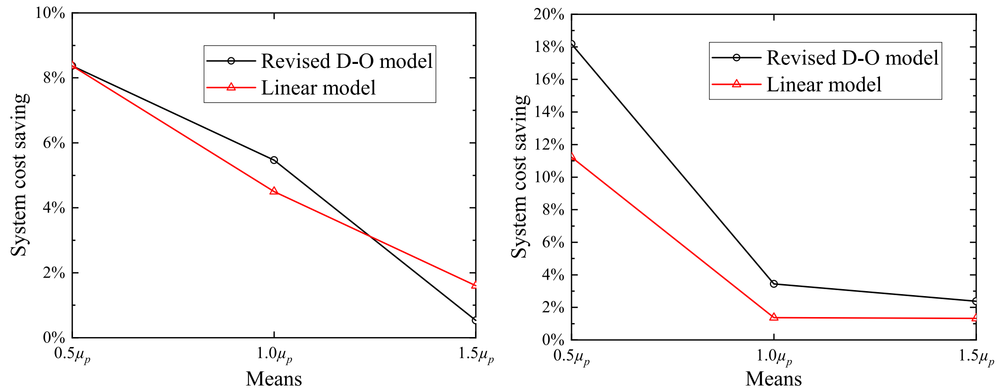
5.3. The likelihood of $o^*(\tau)$ taking vehicle capacity

This section checks the effects of the vehicle capacity constraint in Equations (6) and (13). We show how often the boundary value (C) is taken in the original and revised D-O models, leading to a reduction to the hard-target one. The results imply to what extent the optimizations of soft-target parameters are compromised. Note that the linear model is not included in this examination since the vehicle capacity constraint is not binding in all tested scenarios.

Figure 7 plots the likelihood of $o^*(\tau) = C$, expressed by the percentage of simulation instances that dispatch upon the target of vehicle capacity. As seen, the revised D-O model



(a) Scenarios with different STDs: normal distributions (left); gamma distributions (right)



(b) Scenarios with different means: normal distributions (left); gamma distributions (right)

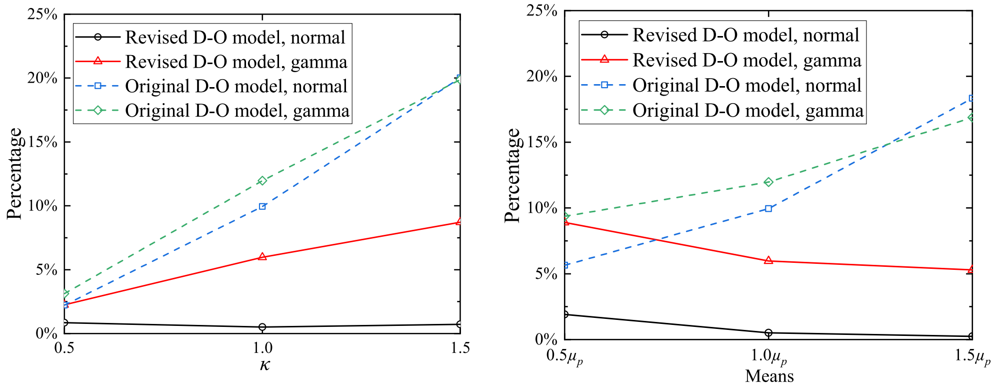
Figure 6. System cost saving of two soft-target models compared with the original D-O model. (a) Scenarios with different STDs: normal distributions (left); gamma distributions (right) and (b) Scenarios with different means: normal distributions (left); gamma distributions (right).

produces less percentage compared to the original one over all scenarios. The percentage reduction of our model is enhanced under demand distributions with larger STDs.

5.4. Impact of passengers' VOT

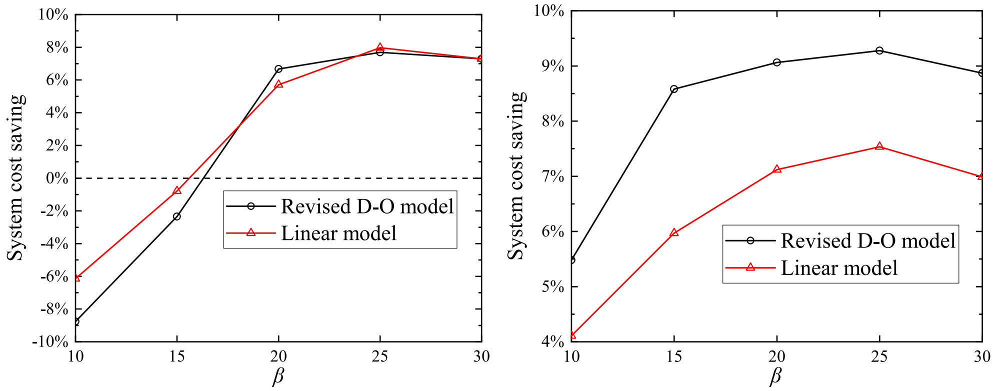
We examine the impact of passengers' VOT by adopting $\beta \in \{10, 15, 20, 25, 30\}$ [\$/(hour·passenger)], representing low to high VOTs in poor and wealthy cities. The results are plotted in Figure 8.

The proposed models perform best for passengers with medium VOTs, but show decreased performance with low and high VOTs. The results can be explained as follows. For low VOTs, the hard-target model (Equation (4)) tends to dispatch vehicles with a full occupancy C, whereas soft-target models may have lower occupancy targets, causing more frequent dispatches and additional operational costs than the hard-target one. As for large VOTs, both hard- and soft-target models tend to dispatch with short headways, and the difference among them becomes smaller.

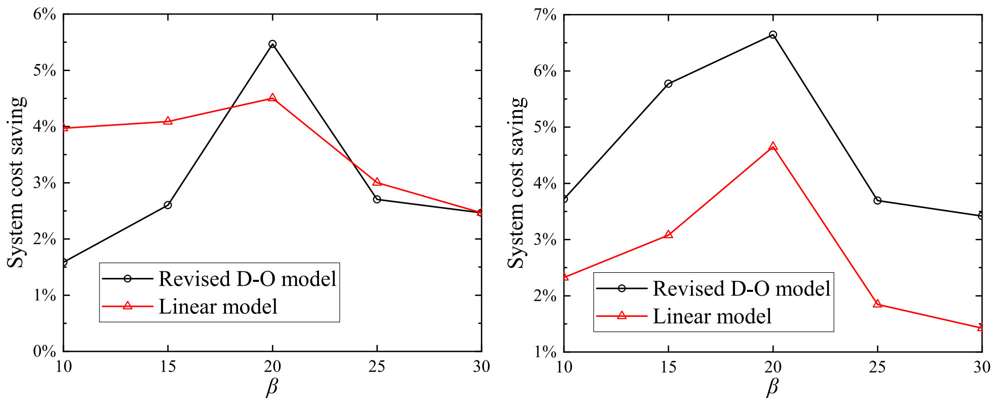


(a) Scenarios with different STDs (b) Scenarios with different means

Figure 7. Percentage of instances that take the vehicle capacity as the target. Linear model is not included since the vehicle capacity constraint is not binding, i.e. its target strictly decreases from vehicle capacity. (a) Scenarios with different STDs and (b) Scenarios with different means.



(a) Compared with hard-target model



(b) Compared with original D-O model

Figure 8. Impact of changes in passengers' VOT under normal distribution (left) and gamma distribution (right) with default means μ_p and STDs $\sigma_p = \mu_p$. (a) Compared with hard-target model and (b) Compared with original D-O model.

Table 7. Fleet size required under soft- and hard-target models [vehicles].

Demand			Hard-target model		Original D-O model		Revised D-O model		Linear model	
mean	STD	VOT	normal	gamma	normal	gamma	normal	gamma	normal	gamma
μ_p	$0.5\mu_p$	20	6	7	7	7	7	8	7	9
	$1.0\mu_p$		10	10	10	11	11	12	10	13
	$1.5\mu_p$		12	11	12	12	13	13	13	13
$0.5\mu_p$	$1.0\mu_p$	20	8	9	8	10	9	12	9	12
$1.0\mu_p$			10	10	10	11	11	12	10	13
$1.5\mu_p$			11	11	11	11	12	13	13	15
μ_p	$1.0\mu_p$	10	7	8	6	11	10	12	9	12
		15	9	7	7	11	11	12	9	12
		20	10	10	10	11	11	12	10	13
		25	10	11	10	13	12	14	12	13
		30	12	11	10	13	14	15	14	15

It's worth noting that while the suggested models consistently outperform the original D-O model, they may be surpassed by the hard-target model for $\beta \le 15$ [\$/(hour·passenger)] (or equivalently higher dispatch cost c_f) under the demand of normal distributions. As a result, the suitability of the aforementioned dispatch models is contingent on local socio-economic conditions.

5.5. Considering fleet size cost

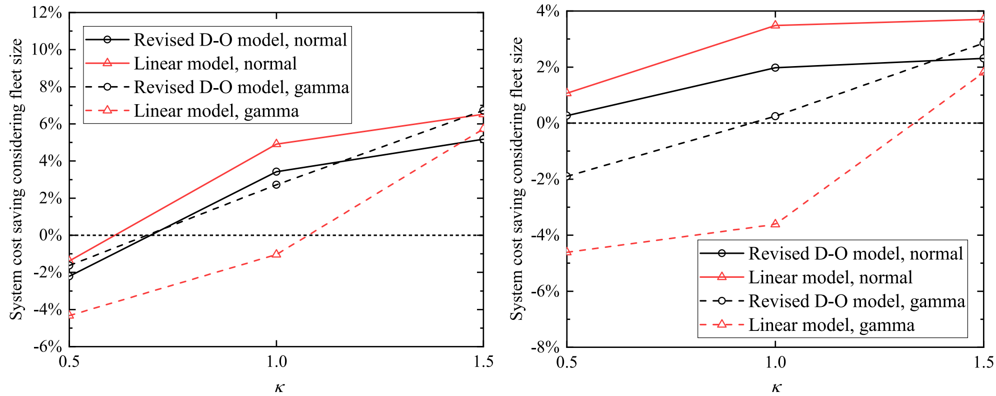
The proposed models increase the number of dispatches, which may require extra fleet size. Table 7 summarises the required fleet size for all test scenarios. This result might cause additional costs to the operator and compromise the advantage of our models. In practice, however, a transit operator often operates multiple lines, among which vehicles can be shared for maximum efficiency. Moreover, operators generally hold spare vehicle reserves to ensure service reliability during peak demand.

This section conducts a sensitivity analysis setting that operators purchase additional vehicles exclusively due to the implementation of our proposed models. The vehicle purchase cost is converted into per-hour units, and is calculated using the discount formula (see footnote 1). The following parameter values are adopted: purchase cost (e.g. BYD bus) is 300000 [\$/vehicle], $d_r = 6\%$, $L_s = 10$ [years] and annual operating hours $= 300 \times 15 = 4500$ [hours].

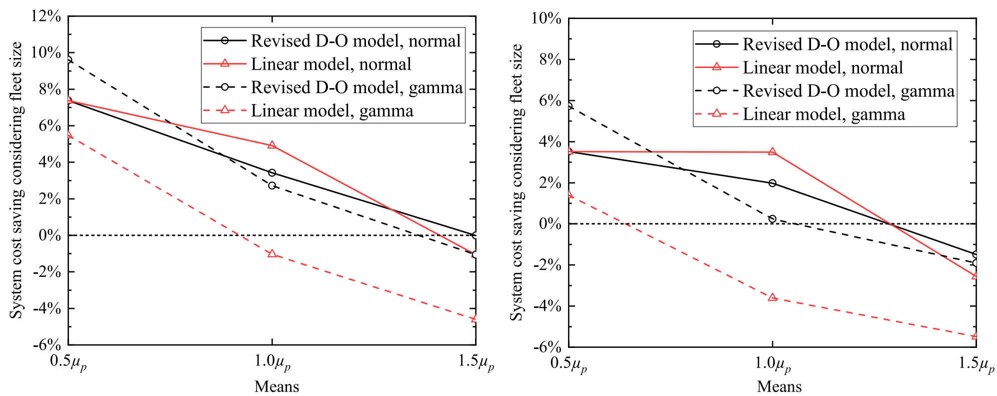
Figure 9 presents the results of systems cost saving for the two soft-target models considering fleet size cost. We observed that our models still remain their advantage under the demand distribution with small means and large STDs (Figure 9(a,b)) or medium VOTs (Figure 9(c)). Unfortunately, in some scenarios (e.g. large means or small STDs), due to the extra fleet size cost, there is no advantage.

5.6. Real case study

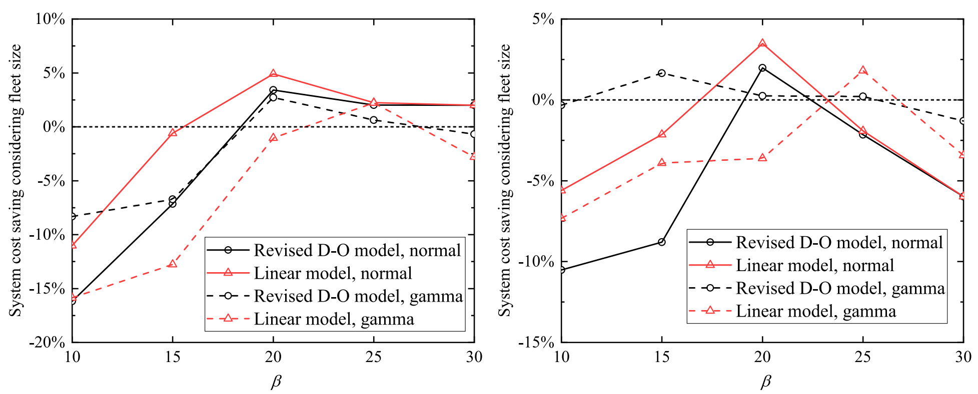
This section demonstrates an application of the proposed models to a shuttle line connecting Daxing International Airport and Tongzhou district in Beijing, China, as shown in Figure 10. The length of the shuttle line is 56.7 km, and the average cycle (round-trip) time is 2 hours.



(a) Scenarios with different STDs: compared with hard-target model (left) and original D-O model (right)



(b) Scenarios with different means: compared with hard-target model (left) and original D-O model (right)



(c) Scenarios with different VOTs under default means μ_p and STDs $\sigma_p = \mu_p$: compared with hard-target model (left) and original D-O model (right)

Figure 9. System cost saving of two soft-target models compared with hard-target and original D-O model considering fleet size cost. (a) Scenarios with different STDs: compared with hard-target model (left) and original D-O model (right). (b) Scenarios with different means: compared with hard-target model (left) and original D-O model (right) and (c) Scenarios with different VOTs under default means μ_p and STDs $\sigma_p = \mu_p$: compared with hard-target model (left) and original D-O model (right).

Figure 10. Shuttle line between Daxing International Airport and Tongzhou district (Beijing) (source: Google Map).

This shuttle line currently adopts a hard-target dispatch strategy with o set to be the vehicle capacity $C = 16$ [passengers per vehicle], and a maximum headway of 30 minutes. The operational horizon is from 7:30 to 22:30, which is uniformly divided into three periods in simulation, i.e. 7:30–12:30, 12:30–17:30, 17:30–22:30, representing the morning-peak ($p = 1$), off-peak ($p = 2$), and evening-peak ($p = 3$) periods, respectively.

We obtained demand data for three months (ranging from July 1, 2020, to September 30, 2020), provided by the Beijing Traffic Operation Monitoring and Dispatch Center and Airport Information Interaction Platform (B. Yang et al. 2022). We calibrate and find the best-fit distribution through the Kolmogorov–Smirnov (KS) test. The demand in all three periods conforms to the gamma distribution. The means and STDs of passengers' arrival rates are $[\mathrm{E}(\lambda_1), \mathrm{E}(\lambda_2), \mathrm{E}(\lambda_3)] = [200, 90, 150]$ [passengers/hour] and $[\mathrm{STD}(\lambda_1), \mathrm{STD}(\lambda_2), \mathrm{STD}(\lambda_3)] = [44, 20, 45]$ [passengers/hour], respectively. The optimised model parameters are $\alpha^* \in [0.01, 1.00, 0.27]$ and $\gamma^* \in [15.50, 1.50, 7.56]$ for the revised D-O model and $\theta^* \in [0, 34, 15]$ for the linear model. Other parameters (e.g. β and c_f) are kept the same as in the above numerical studies.

Table 8 summarises the results, including the CV and mean of headway, total numbers of dispatches, expected generalised system cost (Z), the likelihood of $o^*(\tau) = C$, the fleet size, and fleet size cost. Note that the fleet size is obtained by the maximum number of dispatches within a cycle time during the three periods. As seen from Table 8, the current practice can be further improved by any of the aforementioned models. Particularly, more benefits are observed from the proposed models with up to 15% savings in average headway (waiting time) and 3.43% in generalised system cost. This outcome might not appear substantial, but it is also not insignificant. The collective savings from daily operations could

Table 8. Results of case study.

Metrics	Current practice	Hard-target model	Original D-O model	Revised D-O model	Linear model
CV(H_i)	0.14	0.14	0.10	0.12	0.13
E(H_i) [seconds]	386	334	332	328	330
Num. of dispatches	138	150	153	151	149
Z [$]	291	287	285	281	282
Likelihood of $o^*(\tau) = C$	99%	45%	52%	48%	47%
Fleet size [vehicle]	17	21	22	21	20
Fleet size cost [$/hour]	26	32	33	32	30

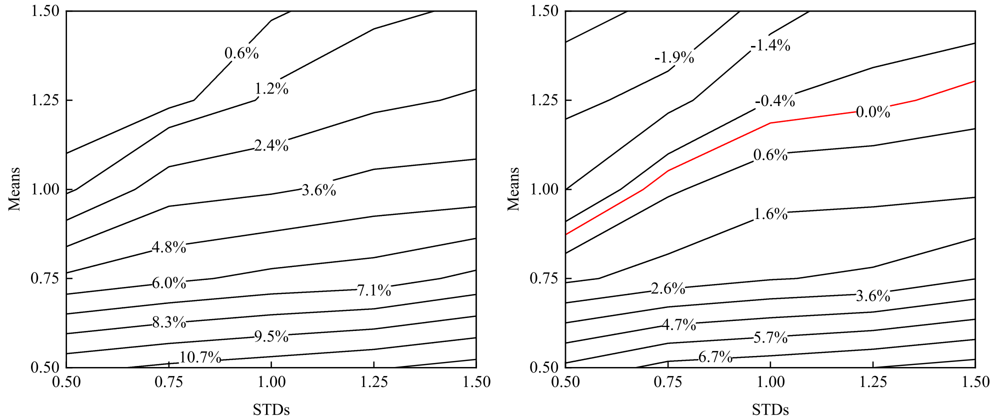
Table 9. Case study results with 17-vehicle cap under two constraint methods: waiting vehicle return and reduced β value.

Setup	Metrics	Current practice	Hard-target model	Original D-O model	Revised D-O model	Linear model
waiting vehicle return	CV(H_i)	0.14	0.15	0.07	0.15	0.15
	E(H_i) [seconds]	386	354	362	398	394
	Num. of dispatches	138	152	142	135	137
	Z [$]	291	292	288	284	287
	Likelihood of $o^*(\tau) = C$	99%	51%	90%	95%	96%
	Vehicle shortage [%]	0%	5.5%	6%	4.5%	5.7%
	Extra delays due to no vehicle [sec/times]	0	103	81	112	110
$\beta = 10$	CV(H_i)	0.16	0.16	0.14	0.15	0.16
	E(H_i) [seconds]	384	386	384	388	386
	Num. of dispatches	139	137	137	136	137
	Z [$]	218	216	216	215	216
	Likelihood of $o^*(\tau) = C$	99%	99%	91%	98%	99%
	Fleet size [vehicle]	17	17	17	17	17

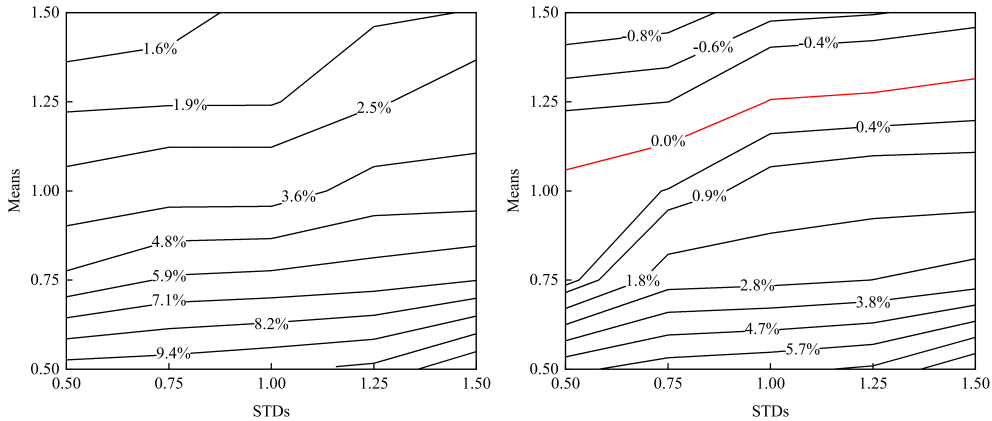
be considerable for the whole society. Potential extra advantages could be generated by induced demand stemming from the enhanced level of service (in the form of reduced waiting time and CV), which is not taken into account in the current study to maintain a conservative estimation.

It is worth noting that the required fleet size is slightly increased in the proposed models as compared to the current practice. In practice, however, vehicles can be shared among multiple lines, and there is an additional reserve of vehicles available. We still believe that operators purchase additional vehicles exclusively due to the implementation of our proposed models. Following the industry practice (e.g. Chevrolet Express), vehicle purchase cost is 50,000 [$/vehicle]. After considering the fleet size cost, the system cost savings decreased from 3.4% to 1.6%. Further, we evaluate two solutions to meet the strict constraints imposed by the operators on fleet size. Table 9 summarises the performance of our proposed models under two scenarios: (i) waiting vehicle return and (ii) reducing VOT. For method (i), the proposed models yield 2.4% system cost savings, yet there will be additional waiting time for passengers due to vehicle shortage (the proportion of vehicle shortage reaches 5.7%). For method (ii), the proposed models yield 1.4% system cost savings in the low $\beta = 10$ scenario. The average waiting time for passengers has slightly increased, and the linear model will become a hard-target model.

To obtain the models' performance boundaries, Figure 11 illustrates the performance of the proposed model relative to the current practice under scenarios involving the expansion and contraction of actual demand means and STDs. When the fleet size is sufficient, our models always perform well. The models exhibit optimal performance when the demand



(a) System cost saving of revised D-O model under not considering (left) and considering (right) fleet size cost



(b) System cost saving of linear model under not considering (left) and considering (right) fleet size cost

Figure 11. Performance of two soft-target models compared with current practice under expansion and contraction of the actual demand means and STDs (ranging from 0.5 to 1.5). (a) System cost saving of revised D-O model under not considering (left) and considering (right) fleet size cost and (b) System cost saving of linear model under not considering (left) and considering (right) fleet size cost.

means are scaled down to 0.5, and the STDs are scaled up to 1.5 times their actual value. Moreover, when the means is expanded to 1.5 times, the soft-target models will lose to the current practice due to the additional vehicle cost under a limited fleet size. Given that real-world statistical data are often subject to errors, this result reveals the models' applicable conditions and provides guidance for operator decision-making.

6. Discussion

The proposed models in this paper is a convex programming problem, which can be solved via the grid search method. It guarantees global optimality with a computational overhead of less than 30 seconds (see Table 3). The proposed two forms of time-declining function have at most only 2 parameters. We believe that even in more complex scenarios, the global optimal solution can still be obtained through the grid search method. While grid

Table 10. Pros and cons of the proposed models.

Model	Pros	Cons	Applicable scenarios
Revised D-O model	✓Avoid prolonged delays ✓Capacity constraint	✗Extra vehicle	✓Gamma (better) and normal ✓Low means and large STDs ✓Moderate VOTs
Linear model	✓Avoid prolonged delays ✓Capacity constraint	✗Extra vehicle	✓Normal (better) and gamma ✓Low means and large STDs ✓Moderate VOTs
Hard-target model	✓Capacity constraint ✓Fewer vehicle	✗Extra delays	✓High means and small STDs ✓Low VOTs
Original D-O model	✓Avoid prolonged delays ✓Simpler operation	✗No capacity constraint	✓High means and small STDs ✓Low or high VOTs

search guarantees global optimality, its computational complexity grows exponentially with problem dimensionality, e.g. more complex target functions with multiple parameters. Future large-scale implementations may leverage gradient-based or Bayesian optimisation techniques as computationally efficient alternatives to grid search.

In the numerical analysis and case study of this paper, the proposed models are tested in a non-reservation-based shuttle bus system. The soft-target model can eliminate the oversaturation and undersaturation problems in systems. In practice, vehicle may depart before reaching full occupancy (the decline of the occupancy target triggers departure) when there is no passenger at the platform, or it may depart at full occupancy, resulting in potential passenger waiting delays at the platform. This operational strategy with unknown schedules may reduce the passenger experience. The operators can display the changes in occupancy target and the number of passengers in real time, enabling the arriving passengers to have an expectation of departure time. This should be considered when making policy-related decisions in actual operation.

Furthermore, our models rely on the VOT of local passengers and the cost of vehicle dispatch. With the widespread application of Automatic Vehicle Location (AVL) and Automatic Passenger Counting (APC) systems, statistical distributions of demand can be obtained at different periods (C. Zhang et al. 2025; Hadas and Shnaiderman 2012; Yuan et al. 2025). Moreover, based on the vehicle's location, vehicles can be dispatched in real time to be shared across multiple lines. The VOT of passengers can be obtained through the stated preference (SP) survey data (Xumei, Qiaoxian, and Guang 2011). The key difference between the two soft-target models and benchmarks lies in their trade-off between operational costs and passenger time, of which the pros and cons are summarised in Table 10. The operators should, based on actual circumstances, analyze the distribution of demand at different time periods and reasonably select the soft-target models.

7. Conclusions

This paper presents two new soft-target models, i.e. a revised D-O model and a linear model, to dispatch vehicles adaptively to uncertain passenger arrivals. The proposed models address the flaws in the existing soft-target model (i.e. the original D-O model) and improve with the capability of dynamically updating the model parameters according to the changes in demand distributions.

Extensive numerical experiments demonstrate the effectiveness of the proposed models in avoiding oversaturation and reducing excessively long headways and passengers' waiting times. Compared to their counterparts (i.e. the hard-target and original D-O models), the proposed models bring significant system cost savings under demand distributions with large STDs and small means. The performance is, however, influenced by passengers' VOT. The superiority of our models culminates for demand with medium VOTs but diminishes for larger VOTs, and may even overturn for smaller VOTs (e.g. under normal distributions). The sensitivity analysis regarding the impact of fleet size indicates that the proposed models remain superior under low means and large variance conditions due to better user experience, although additional vehicles are required.

The application of the proposed models is demonstrated on an airport shuttle line in Beijing (China), of which the demand distributions are calibrated using real data. In addition to the confirmed benefits in the case study, it is worth noting that an additional fleet is required by the hard- and soft-target models, which implicitly assume a sufficient fleet is available and ignore the associated costs. More caution is needed in the cost-benefit evaluation before putting any dispatch models into practice.

The study has several limitations that are worth addressing in future research. Firstly, the soft-target models in this paper are not exclusive. The proposed linear model introduces a hard truncation in the time-declining function form to avoid zero or negative thresholds, which lacks mathematical smoothness. There may be other forms of target functions that are worth exploring, e.g. continuously differentiable negative exponential functions. Secondly, we assume a sufficiently large fleet size and ignore the cost of purchasing or renting vehicles. Additionally, the vehicles are considered to be identical with the same capacity. These assumptions can be relaxed in future research. Thirdly, it is worthwhile to integrate the proposed models into on-demand transit services, such as the flexible-route transit (Kim, Levy, and Schonfeld 2019), ride-pooling services (W. Fan, Gu, and Xu 2024; W. Fan et al. 2025), and modular autonomous transit (Luo et al. 2025; Z. Chen, Li, and Zhou 2020), where demand exhibits uncertainty in both temporal and spatial dimensions. Finally, extensions to traditional fixed-route transit should take into account the extra expense of monitoring real-time arrivals of passengers at all stations/stops along the route.

Note

1. Following previous studies (X. Yang et al. 2021; Yan et al. 2026), vehicle puchase cost can be converted into a per-hour calculation using formula, Hourly cost $=$ Purchase cost $\times \frac{d_r \times (1+d_r)^{L_s}}{(1+d_r)^{L_s}-1} \times \frac{1}{\text{Annual operating hours}}$, where d_r represents the discount rate and L_s is the lifespan. These parameter values can be obtained through industrial practice.

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Data availability statement

No new data were created or analysed in this study. The operation plan is available at https://daxing-pkx-airport.com/transportation/bus/, and demand data can be obtained from the existing paper (DOI:10.13813/j.cn11-5141/u.2022.0021) for the shuttle line at Beijing airport (China) in the present paper.

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